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MULTICRITERIA OPTIMIZATION OF DYNAMIC CONTROL SYSTEMS

The article deals with the dynamic control system. In such systems, the criterion of the quality of management is a functional defined on its decisions. Extremalization functionals is subject to variations ischesleniya. In the case of multicriterion problems solving variational problems repeatedly usugublyayutsya necessity extremalization vector functionals. To solve this problem developed multiobjective nonlinear scheme of compromise on the basis of rational organization.

It has been shown that one of the disadvantages of the principles of homogeneity is that they are not the "economical". Achieving the next level of the relative loss is often implemented at the cost of a significant increase in their overall level. While developers are particularly interested in saving the total consumption of resources in the management system, the use of the integral optimality principle leads to a sharp difference between the levels of the individual losses.

The nonlinear scheme of compromise offers new scope for solving multicriteria problems in different statements. It becomes particularly desirable in cases where the dynamic control system operates in a wide range of possible variation of the external signals, or when the situation is indeterminate or variable.

Mathematical models were shown in the examples.

Keywords: *control, multicriteria optimization, variational problem, nonlinear trade-off scheme, objective function, nonlinear criterion function, rational organization.*

Introduction

In the synthesis of complex modern dynamic systems, account has to be taken of widely different, often contradictory, conditions on the system performance; i.e., we urgently need to develop a formalized approach to the solution of multicriteria variational problems.

Problem formulation. Given the system of differential equations describing the behavior of the control object

$$\frac{dx}{dt} = f(x, u, x^g, z, t) = f(\cdot), \qquad (1)$$

where $x = x(t)$ is the state vector, $u = u(t)$ is the control vector, $x^g = x^g(t)$ is the vector of given signals, $x^0 = x(t_0)$ is the vector of initial conditions, $x^f = x(T)$ is the final state vector, $z = z(t)$ is the vector of disturbing forces, $t \in [t_0, T]$ is time, and f is the vector function of generalized force.

The external conditions x^g, x^0, x^f and z are specified in some form.

Let the given set of partial performance criteria of control system (1) form the vector

$$I = (I_i)_{i=1}^{n}. \qquad (2)$$

Each of the partial criteria is a functional $I_i = I_i(\cdot), i \in [1, n]$, defined on the solutions of system of differential equations (1) with a control from the class of admissible controls U. The vector of partial criteria is restricted to an admissible domain

$$I \in M(I). \qquad (3)$$

The multicriteria variation problem amounts to finding the extremals

$$\{x^*(t), u^*(t)\}, t \in [t_0, T], u^* \in U, I^* \in M(I), \qquad (4)$$

for which vector functional (2) is optimized. For a practical solution of the problem, special supplementary assumptions need to be made.

Analysis of recent research and publications. We can assume without loss of generality that all the partial criteria require minimization, in which case they can be referred to for brevity as losses accompanying the control process. For each partial criterion $I_i(\cdot) \ge 0$, the upper bound of the variation needs to be known:

$$I_i(\cdot) \le B_i, i \in [1, n]. \qquad (5)$$

Information about these bounds is part of the mathematical description of the object.

There may be restrictions on other control system functionals, which for some reason do not belong to the vector (2); then, it is desirable to widen relation (5) and write instead:

$$\varphi_k(\cdot) \le A_k, k \in [1, s], s \ge n, \qquad (6)$$

where φk(·)≥0 is in general a bounded functional of the control system. We shall extend the concept of "loss" to expression (6): we shall assume that, when a limit Ak is reached with respect to any bounded functional φk, the system operation is equally endangered, and that violation of any of inequalities (6) leads to failure of the system. In short, the admissible domain (3) is mapped by expression (5), or in the widened form, by the system of inequalities (6).

To solve constructively the multicriteria synthesis problem we shall use the method of convolution of the partial criteria, whereby the optimization of vector functional (2) is reduced to minimization of some scalar criterion function $\Phi(I)$.There are two different ways of convolution of the criteria [1,2]. The first consists in choosing heuristically the type of criterion function, which is here called the generalized criterion (it is usually a linear function of the partial criteria, while various measures of closeness to the ideal result may be used). The parameters (coefficients) of the generalized criterion are chosen on the basis of the relative importance of the partial criteria.

The second method consists in using a system of axioms for proving the existence of a criterion function (here it is called the usefulness function) of a special kind. In practice, it is almost always a linear function, though in actual fact the axioms about the independence of the partial criteria are most usually violated, in which case, from point of view of prescriptive theory of usefulness, we are not strictly justified in using a linear form. The coefficients of the usefulness function are determined in the light of information about the preferences of the person making the decision.

If we leave aside the actual ways in which the criterion function is determined, the generalized criterion and the usefulness function can be taken to have the same meaning.

To extend the class of problems considered, we shall make weaker assumptions about the criterion function than are made in the usual approach. We shall assume that $\Phi(I)$ is not known a priori, and that all we can say about it is that it is continuous and has continuous partial derivatives with respect to its arguments in the domain (3). It was shown in [3] that, under these assumptions, it is always possible to write a nonlinear function $\Phi(I)$ in the quasilinear form

$$\Phi(I)=\sum_{i=1}^{n}\gamma_i(I)\cdot I_i, \qquad (7)$$

where $\gamma i(I)$ are variable coefficients, forming the continuous vector function $\gamma(I)=\{\gamma_i(I)\}_{i=1}^{n}$.

In fact, let

$$J(I)=\left\| \frac{\partial\Phi}{\partial I_1}\ \frac{\partial\Phi}{\partial I_2} \cdots \frac{\partial\Phi}{\partial I_n} \right\| \qquad (8)$$

be the matrix of partial derivatives of function $\Phi(I)$. If we define

$$\gamma(I)=\int_0^1 J(\xi\cdot I)d\xi, \qquad (9)$$

it can easily be seen that we have the identity

$$\Phi(I)\equiv\gamma(I)\cdot I^T=\sum_{i=1}^{n}\gamma_i(I)\cdot I_i. \qquad (10)$$

By introducing a nonlinear criteria function, we are able to study multicriteria dynamic systems designed for operation in situations with widely variable external signals, or adaptive multicriteria systems, etc.

The absolute value of the function $\Phi(I)$ and of the losses composing it depends both on “internal” causes (the system structure and parameters), and on the external conditions (the disturbing forces, the given signals, the boundary conditions); the collection of these latter characterizes the system operating mode. This mode will be defined as a vector r in the space

$$R=(x^g,x^0,x^f,z)\ni r. \qquad (11)$$

In view of the physical restrictions described above, the vector r will in fact be determined in some domain of space R, i.e., $r\in S(r)\subset R$.

The statement of the multicriteria variational problem is very much dependent on the specification of the system operating mode. In the most complex case, all we know about vector r is that it does not go outside the domain S(r). We arrive at a modified, usually simplified, statement of the problem when the information about the external conditions is refined (statistical [4] or determinate description).

Solution of a complex multicriteria optimization problem may be based on an approach whereby a simpler “basic” problem, typified by fixed external conditions, is solved. Assume that the system is designed for operation in a fixed mode, if we are given the external conditions

$$r=r_0=\{x_0^g,x_0^0,x_0^f,z_0\}\in S(r), \qquad (12)$$

where $x_0^g(t),z_0(t)$ are given determinate functions of time, and x_0^0,x_0^f are the given boundary conditions.

By specifying the external conditions in this way, we can not only simplify solution of the problem as a whole, but we can also linearize the criterion function. In fact, assuming the existence of an optimal solution corresponding to the given external conditions $r=r_o$, we write the expression for criterion function (7) at the optimal point:

$$\Phi^o=\Phi^o(I^o)=\sum_{i=1}^{n}\gamma_i(I^o)\cdot I_i^o. \qquad (13)$$

Since function (7) is assumed to be continuous, we can apply the method of frozen coefficients [5] here, whereby, in the neighborhoods of point Io, the criterion function is described by the approximate equation

$$\Phi(I)\approx\sum_{i=1}^{n}\gamma_i(I^o)\cdot I_i, \qquad (14)$$

where $\gamma_i(I^o)=\gamma_i=\text{const}, i\in[1,n]$, are the “frozen” coefficients. In view of (13), we can see that Eq.(14) becomes exact at the point I=Io.

We can thus draw an important conclusion for the treatment of multicriteria problems, namely, a linear form of criterion function is suitable for the class of given (fixed) external conditions.

In the linear case, the problem of finding the criterion function reduces to determining the constant coefficients γi, $i\in[1,n]$, forming the vector $\gamma=\{\gamma_i\}_{i=1}^{n}$.

We make the assumptions about the vector function f in expression (1) that are usually made in variational problems, namely, that its components are continuous and continuously differentiable with respect to the set of variables x, u, t in their given domain of variation $x,u \in N(x,u)$, $t \in [t0,T]$.

The main material research

Method of Solving "Basic" Problem. In the given mode (12), starting from the object equations (1) and the criterion function in the form (14) with undetermined coefficients γ, we use one of the familiar variational methods for optimizing analytically the control process with $u \in U$. As a result, we obtain an analytic expression for the set of extremals

$$x=x(t,\gamma);\ u=u(t,\gamma), \tag{15}$$

which depend on the unknown coefficients γ.

On substituting (15) in the expression for functionals (6), we obtain

$$\varphi k=\varphi k(\gamma) \le Ak,\ k \in [1,s],) \tag{16}$$

i.e., the losses accompanying the control process are now expressed as functions of coefficients γ. The fact that inequalities (16) are satisfied implies the existence of an admissible domain of variation of the coefficients of the criterion function: $\gamma \in \Gamma\gamma$.

In other words, each extremal of set (15) generates, for $\gamma \in \Gamma\gamma$, in accordance with (16), a specific set of losses, that make up the efficiency vector

$$P(\gamma) = \{\varphi_k(\gamma)\}_{k=1}^{s}. \tag{17}$$

The losses may be of different physical kinds and different dimensionalities. To be able to compare them, we normalize efficiency vector (17) by the constraints vector $A = \{A_k\}_{k=1}^{s}$, and obtain the relative loss vector

$$P_0(\gamma) = \{\frac{1}{A_k}\varphi_k(\gamma)\}_{k=1}^{s},\ \varphi_{0k} \in [0,1]. \tag{18}$$

By analyzing and comparing the components of vector (18) for different values of coefficients $\gamma \in \Gamma\gamma$, we can isolate a vector P0* = P0(γ*) which is in some sense "better" than the rest, and thereby define the vector γ* of required coefficients of the criterion function. Corresponding to these operations we have the formal model of vector optimization

$$\gamma^* = F^{-1}\left[\underset{\gamma \in \Gamma_\gamma}{\text{opt}}\ P_0(\gamma)\right], \tag{19}$$

where F-1 is the inverse of the mapping P0 →γ, and opt is the operator of optimizing the relative loss vector, corresponding to the compromises made in the problem.

We substitute in (15) the optimal values γ* thus found for the coefficients of the criterion function, and hence isolate from this set the required extremals x = x*(t) and u = u*(t).

This describes the general scheme of our analytic method for solving the "basic" multicriteria variational problem.

Discussion of Method. Notice that, in the context of the above scheme, determination of the coefficients of the criterion function, and determination of the extremals of the control process, are both integral parts of a single optimization procedure.

Let us compare our method with another approach to the solution of dynamic multicriteria problems, described in [6, 7]. Here, given the external conditions, the ideal vector Iid is found, while the criterion function is introduced as a measure of closeness to the ideal vector, e.g., in the form of the euclidean norm of the vector I- Iid. This approach demands the solution of n + 1 variational problems, since we have to find the n components of the ideal vector of partial criteria, then extremize a complicated vector functional. In our method, only one variational problem, and that with a functional of conventional type, has to be solved, while all the specific features of multicriteria optimization make their appearance in minimization of a vector function (and not a functional), which is much simpler.

A common feature of the two methods, and indeed, of other methods, is that the method of solution of the multicriteria problem contains heuristic elements. For instance, it is pointed out in [6] that the result of solution depends on the choice of norm in the space of optimized functionals. Similarly, in our method there is the heuristic choice of the scheme of compromises (optimality principle). While the method may contain other heuristic elements (choice of normalization, allowance for priority), the choice of the scheme of compromises is the central feature.

According to one point of view, the heuristic nature of the vector optimization problems is not simply a drawback, but is inherent in multicriteria problems. If we accept this point of view, we must regard as justified the situation in which the result of solving the multicriteria problem depends on the experience and skill of the person solving the problem. While acknowledging the complexity and conceptual nature of these problems, we nevertheless believe that the key trend in vector optimization is the development of methods whereby we can reduce or entirely eliminate the heuristic elements in the final result of solution of the multicriteria problem.

Scheme of Compromises. It is clear from (19) that, for constructive solution of the vector optimization problem, we need to define the scheme of compromises (optimality principle), and thereby reveal the precise sense in which the vector P0* is "better" than the other combinations of relative losses. Mathematically, definition of the scheme of compromises implies expansion of the optimization operator, e.g., in the form

$$\underset{\gamma \in \Gamma_\gamma}{\text{opt}}\ P_0(\gamma) \equiv \min_{\gamma \in \Gamma_\gamma} Y[P_0(\gamma)], \tag{20}$$

where Y is a scalar function of the relative losses.

On the basis of the results of [8, 9], we can write all the optimality principles of practical interest in the unified integral form

$$\underset{\gamma \in \Gamma_\gamma}{\text{opt}}\ P_0(\gamma) \equiv \min_{\gamma \in \Gamma_\gamma} \sum_{k=1}^{s} \varphi_{0k}^{h}(\gamma), h \in [1,\infty]. \tag{21}$$

If we put the formal parameter h equal to unity, we obtain the integral optimality principle

$$\underset{\gamma\in\Gamma_\gamma}{\text{opt}}\ P_0(\gamma)\equiv\min_{\gamma\in\Gamma_\gamma}\sum_{k=1}^{s}\varphi_{0k}(\gamma). \tag{22}$$

If we let h tend to infinity, we obtain the principle of uniformity, equivalent to the Chebyshev model

$$\underset{\gamma\in\Gamma_\gamma}{\text{opt}}\ \equiv\min_{\gamma\in\Gamma_\gamma}\max_{k\in[1,s]}\varphi_{ok}(\gamma). \tag{23}$$

If we take h ∈ [1, ∞], we obtain a range of optimality principles, giving partial equalization of the losses, i.e., providing a solution, intermediate between the two polar schemes of compromises: the Chebyshev operator (23), and the integral optimality principle (22). It is easily shown that the solutions obtained from all the considered schemes of compromises are Pareto-optimal.

Each scheme has its advantages and drawbacks. For instance, operator (23) compels us to minimize the worst (greatest) relative loss, reducing it to the same level as the rest. The relative loss equalization principle

$$\underset{\gamma\in\Gamma_\gamma}{\text{opt}}\ P_0(\gamma)\equiv\{\varphi_{01}(\gamma)=\varphi_{02}(\gamma)=\cdots=\varphi_{0s}(\gamma)\} \tag{24}$$

implies that the most uniform variation of the level of each loss is realized. As an example of this, we may mention a competently designed mechanism that operates uniformly and at the end of its service life, fails simultaneously in each of its sections. It was shown in [10] that, if the solution found by means of condition (24) belongs to the Pareto domain, then it will simultaneously satisfy the min-max principle (23).

One of the drawbacks of uniformity principles is that they are not "economical." The achievement of the closest levels of relative losses is often at the cost of a substantial increase in their overall level.

While application of the integral optimality principle (22) implies that the designer is particularly interested in economizing on total consumption of spares and resources in the control system, the drawback here is that sharp differences become possible between the levels of individual losses.

Example 1. To illustrate the scope of our method, let us take a simple model example. The control object is described by the differential equation

$$\frac{dx}{dt}=-ax+u=f(\cdot), a=\text{const}. \tag{25}$$

The control system is designed for operation in fixed external conditions, and the system operating mode r = r0 = {x0, xf} is characterized by the initial condition x(0) = x0 and the final small neighborhood ε= ±x(T) = ±xf.

The system performance is estimated according to the two partial criteria

$$I_1(\cdot)=\int_0^\infty x^2dt,\ I_2(\cdot)=\int_0^\infty u^2dt. \tag{26}$$

Funcfcionals (26) are defined on the solutions of Eq. (25). The following restrictions are imposed on the functionals (losses) of the system:

1) The required dynamic accuracy is subject to the inequality

$$\varphi1(\cdot)=I1(\cdot)\le A1; \tag{27}$$

2) the energy resources of the system with respect to control are restricted by the inequality

$$\varphi2(\cdot)=I2(\cdot)\le A2; \tag{28}$$

3) the time allowed for realizing the control process is restricted:

$$\varphi3(\cdot)=T\le A3. \tag{29}$$

We pose the problem of finding the extremals of the control process x = x*(t) and u = u*(t) for which the vector functional $I=\{I_i(\cdot)\}_{i=1}^{n=2}$ is optimized.

To solve the problem, we introduce the criterion function F(I), which, since the external conditions are fixed, can be written as F(I) =γ1I1(·)+γ2I2(·). To eliminate the trivial solutions (γ1=γ2=0) we impose the extra condition γ1 = 1. Then, the expression for the criterion function transforms to

$$\Phi(I)=I_1(\cdot)+\gamma I_2(\cdot)=\int_0^\infty\left(x^2+\gamma u^2\right)dt, \tag{30}$$

where γ=γ2.

We use classical variational calculus for finding the extremals without finding γ. On the basis of (25) and (30), we form the Lagrange function

$$L=x2+\gamma u2+\lambda(\dot{x}+ax-u), \tag{31}$$

where λ is the Lagrange multiplier for the nonholonomic connection.

Euler's equations for function (31) are

$$\begin{aligned}&\frac{\partial L}{\partial x}-\frac{d}{dt}\frac{\partial L}{\partial\dot{x}}=2x+a\lambda-\dot{\lambda}=0,\\&\frac{\partial L}{\partial u}-\frac{d}{dt}\frac{\partial L}{\partial\dot{u}}=2\gamma u-\lambda=0.\end{aligned} \tag{32}$$

From the last equation of (32), we find

$$u=\frac{\lambda}{2\gamma} \tag{33}$$

and substitute it in (25). Adding the first equation of (32), we obtain the system

$$\begin{aligned}&\dot{x}=-ax+\frac{\lambda}{2\gamma},\\&\dot{\lambda}=2x+a\lambda.\end{aligned} \tag{34}$$

The characteristic determinant of this system, with root p, is

$$\Delta(p)=\begin{vmatrix}-a-p & \frac{1}{2\gamma}\\ 2 & a-p\end{vmatrix}=0. \tag{35}$$

Hence

$$p=p(\gamma)=\sqrt{a^2+\frac{1}{\gamma}}. \tag{36}$$

If we consider only stable solutions, we can write the expression for the extremals

$$x=C_1e^{-pt},\lambda=C_2e^{-pt}, \tag{37}$$

where C1 and C2 are coefficients that depend on the initial conditions. It follows from the first of (37) that C1 = x0. To find C2, we form the equation

$$(dx/dt)\big|_{t=0} = -C_1 p = -x^0 p = -ax^0 + (C_2/2\gamma). \quad (38)$$

Hence

$$C_2 = 2x^0(a-p)\gamma. \quad (39)$$

Starting from (37) and (33), the general expression for the extremals is

$$x(t,p) = x^0 e^{-pt},\ u(t,p) = x^0(a-p)e^{-pt}. \quad (40)$$

For our future working, it is more convenient to use the function p(y) of (36), rather than the coefficient γ itself; if we know p, we can easily find γ:

$$\gamma=1/(p2-a2). \quad (41)$$

We now obtain the expressions for the relative losses:

$$\varphi01(\cdot)=\varphi1(\cdot)/A1,\ \varphi02(\cdot)=\varphi2(\cdot)/A2,$$
$$\varphi03(\cdot)=\varphi3(\cdot)/A3. \quad (42)$$

On substituting here expressions (40) for the extremals, and using (26)-(29), we obtain the relative losses as functions of p:

$$\varphi_{01}(p) = (1/A_1)\int_0^x x^2 dt = (1/A_1)\int_0^x x^{0^2} e^{-2pt} dt = b(1/p), \quad (43)$$

where $b = x^{0^2}/2A_1$;

φ02(p)=(1/A2)

$$\int_0^x u^2 dt = (1/A_2)\int_0^x x^{0^2}(a-p)^2 e^{-2pt} dt = c(a-p)^2/p, \quad (44)$$

where $c = x^{0^2}/2A_2$.

We find the third loss from the first expression of (40) after substituting in it the boundary conditions

$$|xf| = |x0|\ e\text{-}pT. \quad (45)$$

From this, recalling (29), we have

$$\varphi03(p)=(1/A3)\ln|x0/xf|/p=d/p, \quad (46)$$

where d=(1/A3)ln| x0/xf| .

We now form the relative vector

$$P_0(p) = \{\varphi_{0k}(p)\}_{k=1}^{s=3} \quad (47)$$

and optimize it with respect to the chosen scheme of compromises. Assume that, on the basis of physical considerations, the integral optimality principle (22) has been chosen. Optimization of vector (47) with respect to the integral optimality scheme amounts to minimization of the function

$$Y(p)=\varphi01(p)+\varphi02(p)+\varphi03(p)=b/p+c(a\text{-}p)2/p+d/p. \quad (48)$$

Solution of the equation ∂Y(p)/∂p=0 leads to the result

$$p^* = p^+ = \sqrt{a^2 + \frac{b+d}{c}}. \quad (49)$$

We substitute this result in (40) and thereby extract from the set of extremals the required extremals

$$x=x^*(t)=x0e\text{-}p^*t,\ u=u^*(t)=x0(a\text{-}p^*)e\text{-}p^*t. \quad (50)$$

The coefficient γ* is found from (41), and the coefficients of the criterion function are

$$\gamma1=1,\ \gamma2=\gamma^*. \quad (51)$$

On eliminating time from expressions (50), we obtain the optimal control law

$$u^*=(a\text{-}p^*)x. \quad (52)$$

Nonlinear Scheme of Comprimises. The choice of scheme of compromises needs to be related to the situation for which the optimal solution is being sought; as the situation changes, corrections need to be made to the optimality principle [8, 9, 11]. In the case of dynamic control systems, the "situation" is identified with the "mode," regarded as the set of external conditions for which the system is required to operate.

If the dynamic system is designed for operation in a stressed mode, the implication is that the external signals may be such that one or more losses are in close proximity to their limit. If a loss does reach (or exceed) its limit, it is no compensation that the other losses meantime remain at a low level (by hypothesis of the problem, normal operation of the system is disrupted as soon as any constraint is violated). In such a situation, the increase of the most dangerous loss (i.e., the loss closest to its limit) needs to be held back, without regard to any possible increase in the other losses. In short, the optimization operator most suitable for the stressed operating mode is the Chebyshev operator (23).

On the other hand, if the situation is such that no danger can arise of violating the constraints, it is better to use the "economical" operator (22); this ensures minimal overall losses, while the possible substantial differences in the individual loss levels present no danger in this "quiet" situation. Intermediate modes demand schemes of compromises that offer varying degrees of partial equalization of losses.

The above analysis is by way of being axiomatic: by accepting its recommendations, we can form a unified universal scheme of compromises, which can be adapted to any situation with any degree of stress of the system operating mode.

We shall characterize the degree of stress by the proximity of a relative loss to its limit (unity):

$$1-\varphi0k(\gamma)\in[0;1],\ k\in[1,s]. \quad (53)$$

We shall adopt here a nonlinear scheme of compromises, corresponding to which we have an optimization operator dependent on characteristic (53):

$$\underset{\gamma\in\Gamma_\gamma}{\text{opt}}\ P_0(\gamma) = \min_{\gamma\in\Gamma_\gamma}\sum_{k=1}^{s}[1-\varphi_{0k}(\gamma)]^{-1}. \quad (54)$$

It is clear from (54) that, if a relative loss, e.g., φm(γ) m∈[1, s], starts to come close to its limit (unity), then the corresponding term 1/[1 - φm(γ)] in the minimized sum will increase to such an extent that minimization of the sum reduces to minimization of just this one (worst) term. But this is equivalent to action of the Chebyshev operator (23). On the other hand, if the relative losses are remote from unity, then the action of operator (54) is equivalent to the action of the integral operator (22).

Let us show that our proposed scheme satisfies the condition of Pareto optimality. The proof will use the same technique as in [6]. Given:

1) the set of admissible solutions γ∈Γγ, Γγ is convex in En;

2) the solution with respect to the nonlinear scheme of compromises

$$\gamma^\wedge = \left\{ \gamma^\wedge \left| \gamma^\wedge \in \Gamma_\gamma; \forall \gamma \in \Gamma_\gamma : \sum_{k=1}^{s} \left[1-\varphi_{0k}\left(\gamma^\wedge\right)\right]^{-1} \le \sum_{k=1}^{s}\left[1-\varphi_{0k}(\gamma)\right]^{-1} \right. \right\}; \quad (55)$$

3) the set of Pareto-optimal solutions

$$\Gamma_\gamma^K = \left\{ \gamma© \left| \begin{array}{l} \gamma© \in \Gamma_\gamma; \forall \gamma \in \Gamma_\gamma : \varphi_{0k}(\gamma©) \le \varphi_{0k}(\gamma), k \in \\ \in [1, m < s], \varphi_{0k}(\gamma©) < \varphi_{0k}(\gamma), k \in [m+1, s] \end{array} \right. \right\}. \quad (56)$$

We want to show that $\gamma^\wedge \in \Gamma_\gamma^K$.

Assume the contrary, i.e., the solution γ∧ does not belong to set ΓγK. Then, there is a solution γ--∈Γγ such that

$$\begin{array}{l} \varphi_{0k}(\bar{\gamma}) \le \varphi_{0k}(\hat{\gamma}), k \in [1, m < s], \\ \varphi_{0k}(\bar{\gamma}) < \varphi_{0k}(\hat{\gamma}), k \in [m+1, s]. \end{array} \quad (57)$$

In this case, with $\bar{\gamma} \in \Gamma_\gamma$, we have the inequality

$$\sum_{k=1}^{s}\left[1-\varphi_{0k}(\bar{\gamma})\right]^{-1} < \sum_{k=1}^{s}\left[1-\varphi_{0k}(\hat{\gamma})\right]^{-1}, \quad (58)$$

which contradicts the definition of solution with respect to the nonlinear scheme of compromises (55). Consequently, $\gamma^\wedge \in \Gamma_\gamma^K$.

Discussion of Nonlinear Scheme. Formally, the scope of the range of schemes of compromises for the operator (54) is similar to that for the unified Integral scheme (21); but the parameter h in (21) is not linked with the situation and has to be specified heuristically.

The nonlinear scheme of compromises has the property of being continuously self-correcting as the situation (mode) varies. In stressed situations its effect is equivalent to the action of the Chebyshev (min-max) operator, while in quiet situations its action is equivalent to that of the integral optimality operator, and in intermediate situations it gives varying degrees of partial loss equalization. From this point of view, the traditional schemes of compromises can be regarded as the result of "linearization" of the nonlinear scheme at different "working points" (situations). This explains, incidentally, why we call it the nonlinear scheme; in other respects, it is no more nonlinear than, say, the unified integral form (21).

It must be emphasized that the self-correction of the nonlinear scheme according to the situation takes place continuously. Leaving aside the unified integral form (21), which is very difficult to apply in practical situations, the traditional procedure for choosing the scheme of compromises is realized discretely. This means that, to the subjective errors in solving a multicriteria problem, are added errors connected with the quantization of the schemes of compromises. By using the nonlinear scheme, we can improve the accuracy of solving the "basic" multicriteria problem, thanks to the continuity of the self-correction.

It cannot be said that the use of a nonlinear scheme of compromises entirely eliminates heuristic elements from the process of solving a multicriteria problem. First, there is something heuristic in the acceptance of the axioms implied in our above analysis of the link between situation and choice of optimality principle. And second, relation (53) is not the only way of characterizing stress, nor is the form (54) of the optimization operator unique. Nevertheless, the nonlinear scheme does reduce the subjective errors implied if the situation has to be taken into account when choosing a scheme adequate to the external conditions.

The nonlinear scheme offers new scope for solving multicriteria problems in different statements. It becomes particularly desirable in cases where the dynamic control system operates in a wide range of possible variation of the external signals, or when the situation is indeterminate or variable.

Example 2. In the conditions of Example 1, let us optimize vector (47) with respect to our nonlinear scheme of compromises. Assuming that the solution is reached inside the given domain of restrictions, we solve the equation

$$\frac{\partial}{\partial p}\sum_{k=1}^{s}\left[1-\varphi_{0k}(p)\right]^{-1} = 0. \quad (59)$$

Recalling the notation (43), (44), (46), we can transform Eq. (59) to

b/(p-b)2 + c(a2-p2)/[p-c(a-p)2]2+d/(p-d)2=0. (60)

The required coefficient p* = p∧ is found as the real root of Eq. (60).

Suppose, for instance, that the system is characterized by the numerical data (we omit the dimensions)

a=1; A1=50; A2=100; A3=3; xf=1, (61)

while the initial condition can take values in the range x0 ∈ [0, 15.5].

By using our nonlinear scheme of compromises, we can uniquely solve the "basic" multicriteria problem for any stress properties of the situation (mode). In fact, let us find coefficient p∧ in an extremely stressed mode, when x0 = 15.

Using the notation of (43), (44), (46), we have in this case

b=152/2·50=2.25;
c=152/2·100=1.12; (62)
d=(1/3)ln15=0.9.

Substituting data (62) in Eq. (60) and solving it, we obtain

p∧| x=15=2.39. (63)

Here, in accordance with (43), (44), (46), the relative losses are

φ01∧| x=15=2.25/2.39=0.94;
φ02∧| x=15=1.12(1-2.39)2/2.39=0.90; (64)
φ03∧| x=15=0.9/2.39=0.38.

For comparison, let us use the Chebyshev scheme of compromises in this stressed mode. The simplicity of the examples allows [9] to be used for realizing the Chebyshev model by the unified integral form (21) with

h≥h0=log s/log (1+εφ), (65)

where εφ is the relative error of finding the loss. We form the function

$$V(p)=\sum_{k=1}^{s}\varphi_{0k}^{h}(p)=\left(\frac{b}{p}\right)^{h}+\left[c\frac{(a-p)^{2}}{p}\right]^{h}+\left(\frac{d}{p}\right)^{h} \quad (66)$$

and obtain the necessary condition for its minimum ∂V(p) /∂p = 0. After differentiation, we obtain

bh+c(a2-p2)[c(a-p)2]h-1+dh=0. (67)

The real root of this equation is the solution according to the Chebyshev uniformity principle p=. We specify εφ = 0.01, and in accordance with (65), get

h0=log 3/log (1+0.01)=110.4. (68)

We take h = 121 and write Eq. (67) in the light of (62):

2.25121+1.12(1-p2)[1.12(1-p)2]121+0.9121=0. (69)

The real root of this equation is

p=| x=15=2.41, (70)

i.e., the results (63) and (70) are virtually the same, as ought to be the case in the stressed mode.

Notice incidentally that Eq. (69) is of high degree and difficult to solve, whereas Eq. (60) can be solved quite easily.

In accordance with (49), the integral optimality principle gives

$$p^{+}\big|_{x=15}=\sqrt{1+\frac{2.25+0.9}{1.12}}=1.95, \quad (71)$$

which implies

φ01+| x=15=1.15, φ02+| x=15=0.52,
φ03+| x=15=0.46. (72)

The integral scheme is thus unusable in this stressed situation, since it forces one loss to go beyond its tolerance, whereas the other losses remain at a fairly low level.

Now take an easy mode, corresponding to the initial condition x0 = 2. After calculations, we find that the coefficient corresponding to the nonlinear scheme of compromises has the value

p∧| x=2=3.85. (73)

On using the integral optimality principle in this quiet mode, we obtain

p+| x=2=3.81, (74)

i.e., the results are very similar in both cases. Use of the Chebyshev scheme in this mode gives

p=| x=2=4.38, (75)

which is substantially different from (73) and (74).

It is interesting to consider how the solutions, obtained on the basis of different schemes of compromises, behave when the situation changes. Let us divide the range x0∈[1.5, 15.5] into several subintervals, and solve for each the "basic" problem with the integral, Chebyshev, and nonlinear schemes. The results are shown graphically in Fig. 1. It can be seen that the solutions p* obtained by the nonlinear scheme are the same, at the ends of the range of initial conditions, as the solutions obtained by the polar schemes: on the left by the integral scheme p+, and on the right by the min-max principle p=. At intermediate points of the range, the curve p*(x0) lies between the p+(x°) and the p=(x0) curves.

It is useful to compare this picture with the corresponding curves of relative loss variation in Fig. 2. It can be seen from the latter, in particular, that if the nonlinear scheme (or the min-max principle) is used in stressed modes, the system can remain operational with values of the initial deviations right up to 15.5, whereas, with the integral optimality principle, the first loss can go beyond its tolerance even with x0 = 14.0. On the other hand, in quiet situations, use of the nonlinear scheme gives the same relative loss distribution as does the integral scheme; this shows that the nonlinear scheme is economical in situations where there is no danger of the system violating its constraints.

Nonlinear Criterion Function. If the multicriteria system is studied in a wide range Ω(r) of external conditions, the criterion function Φ(I) needs to be written as a nonlinear relation in partial criteria.

Let the system operation be considered in q fixed modes, corresponding to each of which we have a vector

rj={xjg,xj0,xjf,zj}∈Ω(r)⊂S(r), j∈[1,q]. (76)

We showed above that, with small deviations from a fixed mode, the criterion function can be linearized and written as

$$\Phi^{j}(I)=\sum_{i=1}^{n}\gamma_{i}^{j}I_{i},j\in[1,q], \quad (77)$$

where γij is the coefficient of the i-th partial criterion in the j-th mode. If the system operates in the strictly designed mode, Eq. (77) is exact.

If we assume that coefficients $\{\gamma_{i}^{(j)}\}_{i=1}^{n}$, $j\in[1,q]$, are known, we obtain as a result of this discussion the q combinations

(Φ(1);I1(1),I2(1),...,In(1)),(Φ(2);I1(2),I2(2),...,In(2)),...,(Φ(q);I1(q),I2(q),...,In(q)), (78)

which can serve as reference points for approximation of the criterion function Φ(I) by the approximating function F(I).

In short, the proposed scheme demands the solution of two problems: determination of the coefficients {γi(j)}i=1n, j∈[1,q], of the linearized criterion function at q fixed modes, and construction of the approximating function F(I). The first problem is solved within the framework of the "basic" multicriteria problem, an important point being that we necessarily have to make use of our nonlinear scheme of compromises, with the property of being continuously self-correcting. The second problem may be solved by least squares.

Example 3. In the conditions of Examples 1 and 2, let the system operating mode r = {x0, xf} be characterized by the final state xf = 1 and an initial condition that can vary in the domain Ω(r) = Ω(x0) = [x0min,x0max] = [1.5, 14.0].

We pose the problem of finding the nonlinear scalar relation Φ = Φ(I), connecting the criterion function Φ with the partial criteria I1 and I2, assuming that the initial conditions can vary throughout their range.

We consider the system operation in 21 modes (q = 21); the results are shown in Table 1. We shall seek the approximating function in the class of second-order interpolation polynomials:

F(I)=β1I1+β2I2+β3I12+β4I22+β5I1I2, (79)

where {βm}m=1s are the unknown coefficients of regression.

In accordance with the method of least squares, the unknown coefficients of the interpolation polynomial are found from the condition for minimizing the sum of the error-squares:

$$E = \sum_{j=1}^{s} \left(\Phi^j - F^j\right)^2. \qquad (80)$$

Using the necessary condition for a minimum: ∂E /∂βm =0, m ∈ [1; 5], we obtain the simultaneous system of normal equations:

$$\begin{aligned}
&\sum_{j=1}^{q} I_1^j \left(\Phi^j - \beta_1 I_1^j - \beta_2 I_2^j - \beta_3 I_1^{j2} - \beta_4 I_2^{j2} - \beta_5 I_1^j I_2^j\right) = 0,\\
&\sum_{j=1}^{q} I_2^j (\text{idem}) = 0,\\
&\sum_{j=1}^{q} I_1^{j2} (\text{idem}) = 0, \qquad (81)\\
&\sum_{j=1}^{q} I_2^{j2} (\text{idem}) = 0,\\
&\sum_{j=1}^{q} I_1^j I_2^j (\text{idem}) = 0.
\end{aligned}$$

Substituting here the numerical data of Table 1, and solving system (81) for βm}m=15, we obtain

Φ(I)≈F(I)=0.6245I1+0.1719I2+0.1430I12+0.0045I22-0.0535I1I2. (82)

Principle of Rational Organization. To perform the control function successfully, given the operating conditions, any system must have certain (in general, limited) margins and resources (in the sense of strength, temperature, amount of fuel, etc.). In the usual statement, the limits of the margins and resources are regarded as fixed and given. But cases are common in practical synthesis of multicriteria systems (especially at the early design stages), in which the designer has some scope for varying some or all of the margins and resources, and selecting a set of limits for them which is in harmony with the given external conditions.

Every scheme of compromises reflects a quite specific useful property which the designer deems to be desirable for the system in the considered situation. If the solutions obtained on the basis of different schemes of compromises are the same (or nearly the same), this implies that the margins and resources are chosen and utilized so successfully that, in the given conditions, the system simultaneously meets all the demands made in the different optimality principles, i.e., the system is rationally organized.

When the solutions are identical, the problem of selecting the scheme of compromises falls out, and the heuristic element disappears from solution of the multicriteria problem. The problem of vector optimization then reduces completely and objectively to a problem of scalar optimization.

The principle of rational organization in multicriteria problems may be stated as follows: in the rationally organized system, given the operating conditions, the restricted margins and resources are chosen in such a way that optimization of the efficiency vector with respect to different schemes of compromises leads to identical (or almost identical) solutions.

Since the principle of rational organization is universal and can be used for practical solution of a wide variety of multicriteria problems, we shall develop in a quite general form the constructive apparatus for realizing the principle.

Given the set of admissible solutions Γ⊂En in which are defined the vectors x = {xi}i=1n of n-dimensional Euclidean space. The quality of a solution is estimated from a set of local criteria, represented by scalar functions y1(x), y2(x),...,ys(x). The local criteria form the s-dimensional efficiency vector y ={yk}k=1s defined in the set Γ. We can assume without loss of generality that all the local criteria require minimization (in which case we can briefly refer to them as losses). We know that the losses are bounded: 0 ≤ yk(x) ≤Ak, k ∈ [1, s], though the concrete values of the bounds Ak are not defined and may be chosen from some given admissible set Γa of the constraints vector A = {Ak}k=1s.

We pose the problem: 1) of finding the optimal solution x* belonging to Γ and optimizing the efficiency vector y; 2) of finding the optimal constraints vector A* ∈ Γa, for which the principle of rational organization is satisfied.

We can assert that, if the principle of rational organization is satisfied, then the optimal solution will belong to the Pareto domain

ΓK={x*| x*∈Γ;∀x∈Γ:yk(x*)≤yk(x),k∈[1,m<s];yk (x*)<yk(x),k∈[m+1,s], (83)

it will simultaneously satisfy all the schemes of compromises, leading to Pareto-optimal solutions, and it will be unique.

Hence it follows that, mathematically, realization of the rational organization principle is none other than degeneration of the Pareto domain ΓK to a single point x*, which is the required optimal solution of the multicriteria problem.

We normalize the efficiency vector by the constraints vector and obtain the relative loss vector

$$y_0 = \left\{\frac{1}{A_k} y_k(x)\right\}_{k=1}^{s} = \{y_{0k}(x, A)\}_{k=1}^{s}. \qquad (84)$$

Assuming that the convexity conditions hold, under which Carlin's lemmas are valid [12], we can write the expression for the domain of compromises (83) as the solution of the parametric programming problem

$$\Gamma^K = \bigcup_{\alpha\in\Gamma_\alpha} F^{-1}\left[\min_{x\in\Gamma}\sum_{k=1}^{s}\alpha_k y_{0k}(x,A)\right], \quad (85)$$

where F-1 is the inverse of the mapping y0→x, α={αk}k=1s is a vector parameter, defined in the set

$$\Gamma_\alpha = \left\{\alpha \left| \sum_{k=1}^{s}\alpha_k = 1; \alpha_k > 0 \right.\right\}. \quad (86)$$

Since, when the principle of rational organization is satisfied, the Pareto domain ΓK contracts to the point x*, expression (85) must transform to

$$x^* = \bigcup_{\alpha\in\Gamma_\alpha} F^{-1}\left[\min_{x\in\Gamma}\sum_{k=1}^{s}\alpha_k y_{0k}(x,A)\right]. \quad (87)$$

Since the point x* is unique, the sum $\sum_{k=1}^{s}\alpha_k y_{0k}(x,A)$ in (87) must be invariant with respect to the parameters α∈Γα. This sum is only independent of these parameters if

y01(x,A)= y02(x,A)=...= y0s(x,A). (88)

For, if the relative losses are equal, y01 = y02 = . . . = yos = μ, then, in view of property (86) of the vector parameter $\sum_{k=1}^{s}\alpha_k = 1$, the sum takes the form

$$\sum_{k=1}^{s}\alpha_k y_{0k} = \sum_{k=1}^{s}\alpha_k\mu = \mu\sum_{k=1}^{s}\alpha_k = \mu\cdot 1 = \mu \quad (89)$$

and is independent of the parameters α∈Γα.

On the other hand, the unique point x* must belong to the Pareto domain with any set of parameters α∈Γα. In view of the arbitrariness of the parameters, and property (86), we obtain

α1=α2=...=αs=1/s. (90)

Then, expression (87) takes the form

$$x^* = F^{-1}\left[\min_{x\in\Gamma}\sum_{k=1}^{s}\frac{1}{s}y_{0k}(x,A)\right] = F^{-1}\left[\min_{x\in\Gamma}\frac{1}{s}\sum_{k=1}^{s}y_{0k}(x,A)\right]. \quad (91)$$

We know that a constant factor does not change the position of the extremum of a function; hence we can cancel the factor 1 /s and obtain

$$x^* = F^{-1}\left[\min_{x\in\Gamma}\sum_{k=1}^{s}y_{0k}(x,A)\right]. \quad (92)$$

Degeneration of the Pareto domain ΓK to the single point x* is represented by the intersection of conditions (88) and (92) with μ = 1. Expanding (88), we obtain the system of equations

y0j(x,A)-y0,j+1(x,A)=0, j∈[1,s-1]. (93)

Condition (92) generates the system of equations

$$\frac{\partial}{\partial x_i}\sum_{k=1}^{s}y_{0k}(x,A) = 0, i\in[1,n]. \quad (94)$$

We have to consider (93) and (94) together as a simultaneous system of equations. But this system is clearly indeterminate in the general case, inasmuch as a nondegenerate solution can be obtained with an infinite number of combinations of absolute values of the constraints. In short, we need to complete the definition of the problem with an extra condition, e.g., the condition that the l-th relative loss (or in our present case, all the relative losses) be equal to a given quantity:

y0l(x,A)=μ≤1. (95)

Moreover, in applications one constraint is often given:

Al=Alo (96)

and cannot be varied. In this case, μ is not specified, and instead of condition (95) we have to use (96).

To sum up, to solve a multicriteria problem on the basis of the principle of rational organization, we have to solve the system of equations (93), (94), (95), or (93), (94), (96). As a result, we obtain the n required components of the solution vector x* and the s optimal components of the constraints vector A*.

The above simple and objective method can be used if the problem of rational organization has an exact solution in a given bounded domain of the arguments. If this is not the case, heuristic devices have to be employed. Even then, the principle of rational organization can be utilized constructively (for more details on this point, see [13, 14]).

Example 4. Retaining the other conditions of Examples 1 and 2, and taking x0 = 10, assume that the designer can select the constraints from the given ranges

A1∈[0;50], A2∈[0;200], A3∈[0;3]. (97)

At the end of the control process, we want all the relative losses to take the value μ = 0.5.

We pose the problem of finding, in the context of the principle of rational organization; 1) the extremals xopt(t) and uopt(t) of the control process; 2) the coefficient γopt of the criterion function; 3) the optimal values A1opt,A2opt and A3opt.

We make calculations on the basis of (31)-(46), and obtain expressions for the relative losses. We shall assume, first, that the problem has an exact solution, and second, that this solution is reached inside the given ranges of constraints (97). We form the system of equations (93), (94), (95): in our present example it takes the form

$$\begin{aligned}
&\varphi_{01}-\varphi_{02} = \frac{b}{p} - c\frac{(a-p)^2 p}{} = 0,\\
&\varphi_{02}-\varphi_{03} = c\frac{(a-p)^2}{p} - \frac{d}{p} = 0,\\
&\frac{\partial}{\partial p}(\varphi_{01}+\varphi_{02}+\varphi_{03}) = \frac{\partial}{\partial p}\left[\frac{b}{p} + c\frac{(a-p)^2}{p} + \frac{d}{p}\right] = 0.
\end{aligned} \quad (98)$$

Solving system (98) and substituting the numerical data, we obtain Popt = 3.0, b = d = 1.5; c = 0.375. Recalling the notation in (43), (44), (46), we find that A1opt = 33.3; A2opt = 133.3; A3opt = 1.53. Comparing these values with the constraint ranges (97), we see that our assumptions are valid. The expressions for extremals (40) are

xopt(t)=10e-3t, uopt(t)=-20e-3t. (99)

The value of the coefficient of the criterion function is found from (41):

$$\gamma_{opt} = \frac{1}{p_{opt}^2 - a^2} = \frac{1}{3^2 - 1} = 0.125. \qquad (100)$$

And finally, a check shows that φ01=φ02=φ03=μ= 0.5.

Example 5. Suppose that, from physical considerations, one of the constraints, e.g., A3, is known to be given: A3 = A3o = 1.00, and cannot be varied. In this case μ is not specified; all the other conditions of Example 4 are retained.

To solve this new problem, we form the system of equations (93), (94), (96):

$$\begin{aligned} & b - c(a-p)^2 = 0, \\ & c(a-p)^2 - d = 0, \\ & \frac{\partial}{\partial p}\left[\frac{b}{p} + c\frac{(a-p)^2}{p} + \frac{d}{p}\right] = 0, \\ & A_3 = A_3^o. \end{aligned} \qquad (101)$$

Using the numerical data, solution of this system gives Popt = 3.0; b = d = 2.3; c = 0.575, and hence A1opt = 21.7; A2opt = 87.0.

Since the value popt = 3.0 remains the same as in Example 4, the extremals will be given by (99), and the coefficient γ by (100).

We find the relative losses in this new version by substituting the data obtained in (43), (44), (46); we obtain φ01=φ02=φ03= 0.77.

Conclusions and prospects for further research

The nonlinear scheme of compromises offers new scope for solving multicriteria problems in different statements. It becomes particularly desirable in cases where the dynamic control system operates in a wide range of possible variation of the external signals, or when the situation is indeterminate or variable.

When the solutions are identical, the problem of selecting the scheme of compromises falls out, and the heuristic element disappears from solution of the multicriteria problem. The problem of vector optimization then reduces completely and objectively to a problem of scalar optimization.

The principle of rational organization in multicriteria problems may be stated as follows: in the rationally organized system, given the operating conditions, the restricted margins and resources are chosen in such a way that optimization of the efficiency vector with respect to different schemes of compromises leads to identical (or almost identical) solutions.

Since the principle of rational organization is universal and can be used for practical solution of a wide variety of multicriteria problems, we shall develop in a quite general form the constructive apparatus for realizing the principle.

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МНОГОКРИТЕРИАЛЬНАЯ ОПТИМИЗАЦИЯ ДИНАМИЧЕСКИХ СИСТЕМ УПРАВЛЕНИЯ

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В статье рассматриваются динамические системы управления. В таких системах критерий качества управления представляет собой функционал, определенный на ее решениях. Экстремизация функционалов является предметом вариационного исчисления. В многокритериальном случае трудности решения вариационных задач многократно усугубляются необходимостью экстремизации *векторных функционалов. Для решения данной многокритериальной задачи разработана нелинейная схема компромиссов на основе принципа рациональной организации.*

Показано, что одним из недостатков принципов однородности является то, что они не являются "экономичный". Достижение ближайших уровней относительных потерь часто реализуется ценой существенного увеличения их общего уровня. В то время как разработчики особенно заинтересованы в экономии общего потребления ресурсов в системе управления, применение интегрального принципа оптимальности приводит к резкому отличию между уровнями отдельных потерь.

Нелинейная схема компромиссов предлагает новые возможности для решения многокритериальных задач в различных постановках. Это становится особенно желательным в тех случаях, когда динамическая система управления работает в широком диапазоне возможного изменения внешних воздействий, или когда ситуации являются неопределенными или изменяющимися.

Работа математических моделей показана на примерах.

Ключевые слова: *управление, многокритериальная оптимизация, вариационная задача, нелинейная схема компромисов, целевая функция, нелинейная критериальная функция, рациональная организация.*

БАГАТОКРИТЕРІАЛЬНА ОПТИМІЗАЦІЯ ДИНАМІЧНИХ СИСТЕМ УПРАВЛІННЯ

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У статті розглядаються динамічні системи управління. В таких системах критерій якості управління являє собою функціонал, визначений на її рішеннях. Екстремізація функціоналів є предметом варіаційного обчислення. В багатокритеріальному випадку труднощі рішення варіаційних задач багаторазово ускладнюються необхідністю екстремізації векторних функціоналів.

Для вирішення даної багатокритеріальної задачі використовується нелінійна схема компромісів на основі принципу раціональної організації. Робота математичних моделей показана на прикладах.

Ключові слова: *управління, багатокритеріальна оптимізація, варіаційна задача, нелінійна схема компромісів, цільова функція, нелінійна критеріальна функція, раціональна організація.*

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